Kinetics of blast waves in one-dimensional conservative and dissipative gases

Matthieu Barbier

Department of Ecology & Evolutionary Biology,
Princeton University NJ 08544 USA
E-mail: mbarbier@princeton.edu

Abstract. The blast wave caused by a localized release of energy in a gas has become a textbook hydrodynamics problem since the seminal work of Taylor, von Neumann and Sedov. Yet, it has received very little attention at the kinetic level, which can provide a complementary range of insights: notably, transient regimes and the microscopic structure of the shock front, reduced to a singular boundary in continuum equations. As a first step, we study blast waves in a one-dimensional gas of hard particles. This simple limit helps develop important intuitions pertaining to any type of blast, and it is amenable to kinetic analysis – even with the addition of energy dissipation leading to "snowplow" dynamics, or an inhomogeneous mass repartition (as found in astrophysical systems and granular materials). Furthermore, the conservative case proves to be of remarkable interest, in demonstrating subtle aspects of dimensional analysis and their resolution through microscopic insights: we show that it can effectively behave like a zero-dimensional system, reduced to the shock front, depending on whether a length scale appears in the initial mass distribution.

1. Introduction

During the 1940s, the consequences of a massive explosion in the atmosphere became, as one might expect, an active yet secretive research topic. Numerous physicists – among which Taylor [1, 2], Sedov [3] and Von Neumann [4], under the scrutiny of their respective governments – independently discovered the characteristics of the self-similar blast wave that follows such an explosion. In the following decades, this problem became a textbook example of both dimensional analysis and hydrodynamics, with countless applications to other fluids, including astrophysical and laboratory plasmas, and refinements such as heterogeneous or non-conservative media [5]. Yet, to the best of our knowledge, it is only in the context of granular materials that recent experiments [6, 7] and numerical investigations [8–10] have tackled aspects of this problem from the microscopic point of view, at the level of individual particles. In particular, the internal structure of shock fronts, usually treated as discontinuities in hydrodynamic equations, has only begun to be investigated in a kinetic framework [11–14].

We focus here on a supersonic blast wave in strong shock conditions, i.e. exhibiting a singular front where the high-pressure bulk of the blast meets the surrounding gas: as the speed of matter displacement largely exceeds the speed of sound (or even heat transfer) in the ambient medium, no energy can propagate in advance into the fluid at rest and smoothen the gradients at the boundary. The speed of sound is proportional to the thermal velocity $\sqrt{2\Theta/m}$ given thermal energy Θ and particle mass m, and heat conductivity also depends on Θ. The most representative case of strong shock conditions is therefore a "cold gas" with $\Theta = 0$ outside of the blast region, which will be considered throughout this paper. As self-similar phenomena should generally not depend on microscopic particulars (see the Supplementary Materials for clarification), we later simplify the problem further by considering hard-core interactions and modelling energy dissipation by inelastic collisions with fixed restitution coefficient. However, our results should apply to a broad class of systems, provided their dissipation rate likewise increases with density and temperature.

Previous works have derived scaling laws, such as the time-dependence of the radius of the blast $R(t)$, from the conservation of mass and energy in the elastic case, or radial momentum in the inelastic case [15]. We recall these derivations in general spatial dimension d in Sec. 2, and suggest that conservation laws must be supplemented by kinetic insights to give rise to the proper scaling properties. We then consider the 1D elastic gas in Sec. 3, which is highly singular if the initial mass repartition is either uniform or scale-free. This singularity demonstrates how intricacies of dimensional analysis at the macroscopic level are in fact easily solved from a microscopic viewpoint. It also reveals the specific role of the foremost moving particle (playing in $d = 1$ the role of the shock front in higher dimenions), which is also relevant in dissipative systems. We investigate the latter in Sec. 4 and show that the leading particle controls the dynamics during the logarithmic transient phase. Our characterisation of this intermediate regime agrees with analyses from experiments [6, 7]. We conclude on the question of how momentum conservation shapes the scaling regimes in dissipative, standard conservative and accelerated conservative blasts.

2. Scaling laws

This short section recalls general results on scaling laws in the blast in arbitrary dimension, to which we will later confront our results. As we expect the solution to be self-similar, macroscopic quantities such as the number of particles in the blast $N(t)$, its radius $R(t)$ or its total energy $E(t)$ (in case it is not conserved) will follow simple power laws that may be determined by dimensional analysis. It is noteworthy that the applicability of dimensional analysis is in fact rigorously expressed in group-theoretic terms and expounded upon in the classic work by Barenblatt [16]. See the Supplementary Materials for a brief summary of relevant basic theorems and the terminology of self-similarity.

We first summarize the classic argument most famously given by Taylor [1, 2]. At the macroscopic level, the dynamics are completely determined by the laws of conservation of energy and mass, as the energy liberated by the detonation is then distributed among an increasing number of particles N, with total mass M, within the growing radius R of the blast wave. The only parameters describing this evolution are time t, radius R, total energy E, and mass density ρ_i in the initial state of the gas from which we can deduce M. By dimensional analysis, the evolution of the radius must then obey

$$R(t, E, \rho) = R_0 \left(\frac{Et^2}{\rho_i} \right)^{2/(d+2)} \tag{1}$$

as this is the only combination of the three other parameters that has the dimensions of a length. In more descriptive terms, Taylor expressed this as

$$\frac{M\dot{R}^2}{E} = \text{constant} \tag{2}$$

which represents the fact that a finite fraction of the energy is always involved in coherent motion i.e. as kinetic energy of the outward flow, which must scale quadratically with $\dot{R}$ the velocity of the shock front. The rest of the energy goes into random motion, that is thermal energy, but as long as the blast expands the ratio of kinetic over thermal energy should tend to a finite limit. In general terms, we will put it as

$$\begin{aligned} E(t) &\sim N(t)\dot{R}^2(t) \sim R^d(t)\dot{R}^2(t) \\ &\sim \frac{R^{d+2}(t)}{t^2} \sim t^{(d+2)\delta-2} \end{aligned} \tag{3}$$

assuming $R(t) \sim t^{\delta}$. If energy is conserved, $E(t) = E_i$, we find

$$R(t) \sim t^{2/(d+2)} \qquad \text{(conservative case).} \tag{4}$$

hence Taylor's famous result $R \sim t^{2/5}$ in $d = 3$. However all previous relations hold even if $E(t)$ is not constant, as in the case of inelastic collisions between the particles (or in radiative gases). Then another conservation law must be considered: that of total momentum

$$\Pi(t) \sim M(t)\dot{R}(t) \sim \frac{R(t)^{d+1}}{t} \sim t^{\delta(d+1)-1} \tag{5}$$

so that

$$R(t) \sim t^{1/(d+1)} \qquad \text{(dissipative case).} \tag{6}$$

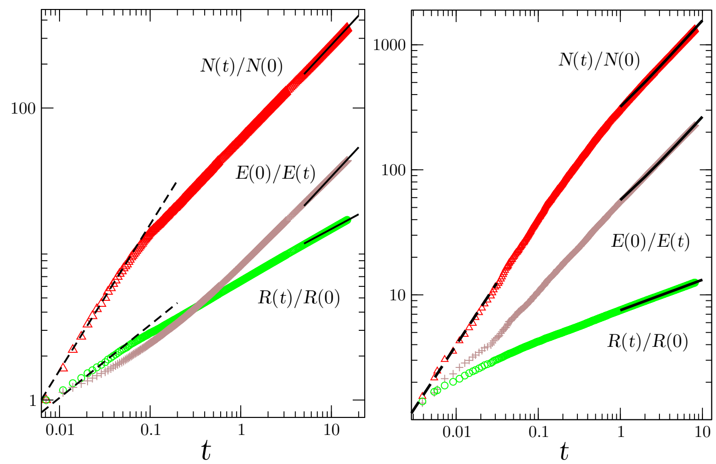

Figure 1. Scaling laws for a dissipative blast in dimension $d = 2$ (left) and $d = 3$ (right) as points of comparison for one-dimensional systems. Particle number N, inverse total energy $1/E$ and radius R rescaled by their initial value (we take the inverse of energy to get an increasing quantity). Solid lines are asymptotes derived from Eqs. (3) and (6), while dashes represent initial ballistic growth. These results are obtained from Hard Sphere Molecular Dynamics simulations, as detailed in the Supplementary Materials.

This behaviour, recently observed in a granular gas context [9], is also well known in astrophysical blasts following Oort's argument [17], and we validated it for multiple values of spatial dimension d using Molecular Dynamics simulations in Fig. 1.

A puzzle immediately arises: why should we consider energy conservation for elastic particles, and momentum conservation for inelastic particles, when both laws would seem to apply to first case? This conundrum is most easily resolved in one-dimensional systems, which are the main focus of this paper (for higher dimensions, see [15]). On the other hand, a system of elastic particles in one dimension exhibits an important singularity, which we must first investigate in Sec. 3 so as to clarify the conditions for the generality of our analysis.

3. Elastic particles

3.1. Singularity of the one-dimensional case

The kinetics of a blast in an homogeneous fluid of elastic hard spheres was first considered a few years ago [8], and we recall it to introduce the inelastic and inhomogeneous cases, but also because its behaviour is strongly singular in dimension $d = 1$; a fact that has not elicited much attention, yet can be understood through dimensional analysis.

A frontal collision between two identical hard spheres simply exchanges their velocities (see the transfer rule (20) in Sec. 4). Collisions in a one-dimensional system are necessarily frontal, and thus particles can be interpreted as merely switching labels on contact. If the initial condition is a "break shot" imparting energy to a single particle, there will always be one particle in motion with constant velocity at any given time. The linear time dependence of this ballistic trajectory disagrees with the law in general dimension given in the previous section:

$$R(t) \sim t \quad \neq \quad t^{2/(d+2)} = t^{2/3}. \tag{7}$$

However, the exponent $2/(d+2)$ derives from the assumption that the proper dimensional parameters in the scaling law for R are t, E_i and ρ_i, where ρ_i is the linear mass of the homogeneous system

$$[\rho_i] = M/L^d = M/L. \tag{8}$$

If there is a single particle in motion, the linear mass is not relevant: the only mass scale involved is the mass m of the particle, and dimensional analysis then naturally entails that velocity should be constant

$$R(t, E_i, m) = \left(\frac{2E_i t^2}{m}\right)^{1/2} \sim t. \tag{9}$$

In fact, having combined microscopic properties such as particle mass m, size σ and mean free path l into a single parameter ρ_i, prior to the dimensional analysis outlined above, was already making an implicit hypothesis on the similarity properties of the system (as we fully demonstrate later). We may reintroduce these parameters in our dimensional analysis, so that there are now 5 parameters for 3 dimensions, hence 2 dimensionless parameters. Let us first consider velocity $\dot{R}$ since, unlike R, it still has only one obvious typical scale that we take as prefactor

$$\dot{R}(t, E_i, m, l, \sigma) = \sqrt{\frac{E_i}{m}}\, \Psi\left(\frac{t}{\tau}, \frac{\sigma}{l}\right) = \sqrt{\frac{E_i}{m}}\, \Psi(\psi_1, \psi_2) \tag{10}$$

with τ the typical collision time for a particle with kinetic energy E_i

$$\tau = \sqrt{\frac{ml^2}{E_i}}. \tag{11}$$

Furthermore, the interval on the line that lies *inside* a particle, i.e. its diameter 2σ, is irrelevant to the dynamics. Therefore we may let it vanish while keeping all inter-particle distances constant (which is exceptional to the 1D case), without affecting any macroscopic quantity, so that

$$\lim_{\psi_2 \to 0} \Psi(\psi_1, \psi_2) = \widehat{\Psi}(\psi_1). \tag{12}$$

Finally, if particles are identical, their collisions have no effect on velocity, so the collision time τ may go to infinity without changing the dynamics

$$\lim_{\psi_1 \to 0} \widehat{\Psi}(\psi_1) = \text{constant} \tag{13}$$

Hence $\dot{R}$ is immediately expressed by its prefactor, and we find the ballistic behaviour

$$R(t, E_i, m, l, \sigma) \propto t\sqrt{\frac{E_i}{m}} \tag{14}$$

with *full similarity* in variables l and σ: they can take arbitrary values without affecting any nondimensionalized function (and here, not even the dimensional observables). In higher dimensions, the same ballistic limit is found if $\psi_1 \to 0$ or $\psi_2 \to 0$, meaning that the collision probability vanishes through either particles being infinitely small or infinitely far apart, but this limit is singular; any finite value of these parameters eventually gives rise to the standard behaviour detailed in Sec. 2.

Another way of reaching similar conclusions is to rewrite the equation so as to involve the number N of particles in motion, which itself is some (as yet unknown) function of the parameters:

$$\dot{R}(t, E_i, m, l, \sigma) = \sqrt{\frac{E_i}{m}}\,\Phi\Big(N, \frac{\sigma}{l}\Big) = \sqrt{\frac{E_i}{Nm}}\,\widehat{\Phi}\Big(\frac{\sigma}{l}\Big) \tag{15}$$

where the second equality comes from conservation of energy over particles with total mass Nm. Then, as we know that any non singular value of the remaining parameter σ/l should not affect scaling exponents, we only have to solve:

- if $N(t) \sim R^d(t)$, $\quad \dot{R}(t) \sim R^{-d/2}(t) \quad \Longleftrightarrow \quad R(t) \sim t^{2/(d+2)}$
- if $N(t) \sim 1$, $\quad \dot{R}(t) \sim 1 \quad \Longleftrightarrow \quad R(t) \sim t$

Hence, expression (15) exhibits *similarity of the second kind* in parameter N (as we needed to solve a differential equation to obtain the exponent) while equation (10) was fully self-similar of the first kind and dimensional analysis was sufficient. This demonstrates that self-similarity may be arduous to detect under the wrong set of variables, and reflects how it hinges on conservation laws: focusing on velocity $\dot{R}$ helps us understand why one should employ the parameter m rather than ρ in the singular case, as it is then the individual momentum $m\dot{R}$ that is conserved. Still, this case-by-case choice of either mass or density in dimensional analysis may seem somewhat arbitrary, until it is replaced within a broader context.

3.2. Spatial mass repartition

There seems to be a straightforward option for removing the singularity of the one-dimensional conservative system: considering particles with different masses, so that collisions distribute energy unequally among partners instead of transferring it entirely to the foremost particle. At least two choices will prove interesting:

- using a mass distribution that is spatially homogeneous when coarse-grained, for instance a binary alternation of masses $m = 1$ and $m = 2$,
- having particle masses decay as a power law with distance to the origin: $m(r) = m_0\, r^{-\omega}$.

The second option is inspired by a common example of self-similar shock in astrophysics: the blast of a supernova in a region of space where, due to gravitational effects, the density of interstellar matter varies with the distance to the center of the star. Setting $\omega = 1 - d_{\text{eff}}$, we may also use it as a one-dimensional analogue to an angular section (e.g. a cone) in higher dimension $d_{\text{eff}} > 1$: instead of the individual mass of a particle, it is the number of particles found between r and $r + dr$ (and thus the total mass of this segment) that increases as a power-law in r. In any case, the usual dimensional argument is easily adapted to find

$$R(t) \sim t^{2/(d+2-\omega)} \tag{16}$$

Results of a Molecular Dynamics simulation, shown in Table 1, allow for a better understanding of the singularity discussed above: binary alternating masses give as expected per Taylor's reasoning (for the standard case $\omega = 0$)

$$\begin{aligned} &R(t) \sim t^{2/d+2} = t^{2/3} \qquad (d = 1) \\ &N(t) \sim R^d(t) \sim R(t) \sim t^{2/3} \end{aligned} \tag{17}$$

Exponent ω	-3	-2	-1	0	1	2	3
Prediction $2/(2+d-\omega)$ for $d=1$	1/3	**2/5**	1/2	**2/3**	1	**2**	∞
Scale-bound	0.33	**0.39**	0.51	**0.65**	1.0	**1.6**	–
Scale-free "$d=0$"	**0.40**	0.50	**0.67**	1.0	**2.0**	–	–

Table 1. Exponents observed for $R(t)$ in Molecular Dynamics simulations for elastic particles on a line, initially at rest except one at $r=0$. The scale-free row corresponds to particles whose masses are distributed according to a power law $m(r) = m_0|r|^{-\omega}$, thus without typical length scale. On the scale-bound row, the masses are similarly computed, then the mass of every second particle is doubled, so that a length scale (the mean free path) appears. As shown by alternating plain and bold face for similar values, the scale-free case is shifted compared to theoretical exponents for $d=1$ and instead corresponds to $d=0$. Asymptotic convergence slows down as $\omega \to d+2$ and numerical testing becomes less reliable.

and the peculiarity of dimension 1 disappears, since the proper dimensional parameter in $R(t)$ is indeed ρ_i. However, a power law mass distribution $m(r) = m_0\, r^{-\omega}$ retains the singularity of the 1D case

$$N(t) \sim R(t) \sim t^{2/(2-\omega)} \tag{18}$$

as deduced from dimensional analysis with mass parameter m

$$R(t, E_i, m_0, \omega) = \left(\frac{E_i t^2}{m_0}\right)^{1/(2-\omega)} \widehat{R}(\omega). \tag{19}$$

Thus, the determining factor for the observed regime is in fact the absence of a typical length scale in the singular 1D system, which is not solved by adding an inherently scale-free spatial mass repartition such as a power law. However the binary alternation adds a length scale which is the separation between two different particles, and the singularity disappears. The microscopic explanation for these different behaviours is that in the scale-free ("zero-dimensional") case, for any choice of ω, only the mass of the leading particle determines the shock propagation speed $\dot{R}(t)$, so that interparticle distances are never relevant.

Interestingly enough, this peculiarity of the foremost particle is magnified if one adds inelasticity to the parameters: we can then find an intermediate asymptotic regime in which the system still exhibits similarity in certain microscopic variables, but also depends on this additional dimensionless parameter, and exhibits exponential and logarithmic rather than power law behaviours.

4. Inelastic particles and momentum conservation

We now investigate the conundrum of scaling and conservation laws in the inelastic case, which we pointed out at the end of Sec. 2. Fortunately, the dynamics of a blast in a one-dimensional system at zero initial temperature can be analysed exactly. For clarity and ease of calculation, let us further impose that all particles are initially equidistant with separation $l = 1$ (although this constraint will be relaxed in simulations).

Different models exist for non-conservative collisions, among which we choose the simplest: the particles are inelastic hard spheres with a fixed restitution coefficient $\alpha \in [0, 1]$, such that $1-\alpha^2$ is the fraction of energy lost in a frontal collision. The interaction rule for two inelastic hard spheres, given their masses m_1 and m_2 and their pre-collisional velocities v_1 and v_2, is given by

$$v_1^* = av_1 + (1-a)v_2$$
$$a = (m_1 - \alpha m_2)/(m_1 + m_2) \tag{20}$$

where v_1^* is the velocity of particle 1 after collision with particle 2, and v_2^* is similarly computed. For equal masses, $a = (1-\alpha)/2$ and elastic particles correspond to $\alpha = 1$. Therefore, if particles are both elastic and identical, $v_1^* = v_2$ and velocities are simply exchanged upon collision. More elaborate models often give α as a function of relative velocities (for a discussion, see for instance [18]). Except for the transient phase, and unless otherwise noted, such refinements do not affect the general content of the following results.

4.1. Transient regime and leader behaviour

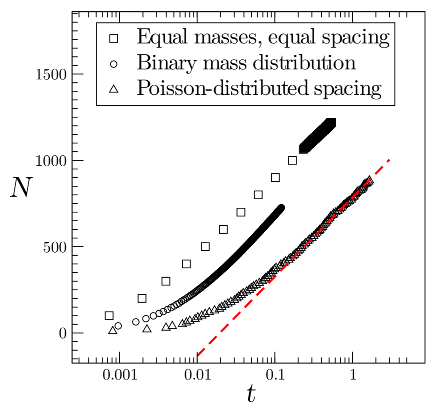

Figure 2. Number of moving particles every 100 collisions up to 30000 in a simulation wth $\alpha = 0.99$. The dashed line corresponds to the asymptotic solution (27) up to an additive constant. The recollision phase is seen for equal masses and spacing as the region where squares cluster.

We observe in Fig. 2 the logarithmic growth of the number of moving particles in the blast. This behaviour is easily understood by the following argument: all particles in the blast are set in motion by the leading particle, whose velocity decreases geometrically (and whose identity is switched) with each collision. As long as none of its followers can catch up with it, the velocity of the shock boundary thus decays exponentially with the distance $\dot{R}(t) \sim e^{-R(t)}$. This can easily be seen in exact calculations. In the initial phase where every particle has gone through a single collision, the $N-1$th contact event gives the following post-collisional velocities

$$v_N^* = \frac{1+\alpha}{2} v_{N-1} \tag{21}$$
$$v_{N-1}^* = \frac{1-\alpha}{2} v_{N-1} \tag{22}$$

so that if $\alpha > 0$, the velocity of the leading particle is always higher than its followers (for $\alpha = 0$ immediate aggregation occurs), hence

$$v_N = \left(\frac{1+\alpha}{2}\right)^{N-1} \tag{23}$$

$$v_k = \frac{1-\alpha}{2}\left(\frac{1+\alpha}{2}\right)^{k-1} \qquad \forall\, k < N \tag{24}$$

representing a trail of particles whose velocity decreases exponentially with their index, see Fig. 3. If α is close to 1, the leader retains a large fraction of the initial momentum for a long time. In that phase, the time needed to reach particle N thus grows exponentially:

$$t_N = \sum_{k=1}^{N-1}\left(\frac{2}{1+\alpha}\right)^k = \left(\frac{2}{1+\alpha}\right)^N \frac{1+\alpha}{1-\alpha} - \frac{2}{1-\alpha} \tag{25}$$

(hence $v_N \propto 1/t_N$). As $N \to \infty$

$$\ln(t_N) \approx \ln\left(\frac{1+\alpha}{1-\alpha}\right) + N\,\ln\left(\frac{2}{1+\alpha}\right) \tag{26}$$

which leads to the asymptote observed in Fig. 2 :

$$N \approx \frac{\ln\left(\frac{1-\alpha}{1+\alpha}\,t_N\right)}{\ln\left(\frac{2}{1+\alpha}\right)} \tag{27}$$

This phase is a case of non-self-similar intermediate asymptotics [16]: energy is no longer a constant parameter, but no other conservation law is applicable instead because none affects solely the leading particle (contrary to the elastic case), which is still entirely responsible for the dynamics; instead, the behaviour depends on a new dimensionless parameter α. Note that α being fixed leads to the above geometric energy loss; a model with velocity-dependent dissipation could lead to e.g. power-laws whose exponents vary with parametric choices. It is only here that a microscopic property may affect the scaling laws, as expected from the lack of global similarity properties in this regime.

4.2. Recollision and scaling law

Eventually, some trail particles become faster than the leader, since the latter's velocity decreases exponentially with N (which at this point is both the number of collisions and the number of particles in motion) while the typical velocity in the trail evolves as $1/N$ and the length of the trail is bounded by N.

Once the leader starts being re-accelerated by collisions coming from behind, and hence receives some of their momentum, the system evolves towards the expected behaviour $N \propto t^{1/2}$ corresponding to the general law $R \sim t^{1/(d+1)}$ in inelastic blasts. This evolution is slowed down by recollision cascades happening in the trail, but the asymptotic power law is reached after the transitory logarithmic phase whose length diverges as $\alpha \to 1$, see Fig. 4. This intermediate logarithmic regime also appears to be the one exhibited by experimental works on bidimensional blasts in granular systems [6, 7].

Thus, it seems that the asymptotic power law emerges from a balance between cooling incurred mostly by the leader, and pressure induced by particles coming

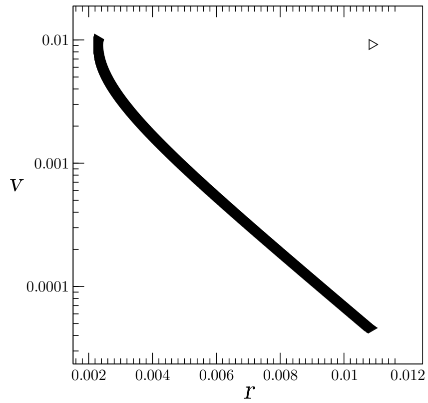

Figure 3. Velocity of moving particles as a function of their position. The leader is isolated with much higher velocity than its direct followers (as $\alpha = 0.99$). The first particles on the left have entered the recollision phase, but the middle section exhibits the exponential decrease with distance predicted for the initial phase.

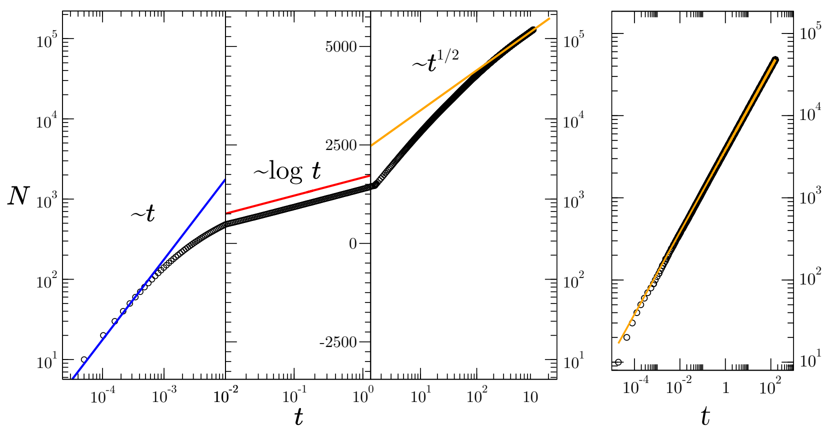

Figure 4. Scaling laws for the number of moving particles $N(t)$. *Left:* With $\alpha = 0.99$, a ballistic linear growth is followed by the logarithmic regime (27) (note the switch to lin-log scale for the middle panel while the other two regimes are plotted in log-log), until the leading particle is accelerated by recollisions coming from the trail and the asymptotic law $N \sim \sqrt{t}$ emerges. *Right:* With $\alpha = 0.5$, the asymptotic law is immediately observed.

from behind. Scaling laws directed by momentum conservation thus correspond to microscopic situations where particles on the inside of the shock wave are pushing those in front of them and new particles constantly accumulate at the front, in agreement with Oort's idea of "snowplow" dynamics [17] which finds here a confirmation at the microscopic level.

Let us finally note that the conservation of momentum is generally a subtle matter, whether particles are elastic or not. In the conservative case,

$$E(t) \sim N(t)\dot{R}^2(t) = E(0) \tag{28}$$

which seems to entail

$$\Pi(t) \sim N(t)\dot{R}(t) \sim 1/\dot{R}(t). \tag{29}$$

However momentum must be conserved over the whole system,

$$\Pi(t) = \Pi(0). \tag{30}$$

If one separates particles into those to the left and to the right of the mass centre (assuming that the initial particle's velocity was toward the right), we may define two quantities Π_- (to the left) and Π_+ (to the right) which can take opposite signs. Their sum must be constant, but each of them may evolve as $1/\dot{R}(t)$ if that is an increasing quantity – for instance, for a mass distribution $m(r) = m_0\, r^{-\omega}$, as

$$\Pi_+(t) \sim -\Pi_-(t) \sim t^{-\omega/(1-\omega)} \tag{31}$$

provided this exponent is positive. As noted in [8], the 1D elastic blast tends to symmetrize like its higher-dimensional counterpart, and one quickly observes $\Pi_+(t) \approx -\Pi_-(t)$. Thus an important flux of momentum occurs between opposite sectors of the blast, going through the centre, and causing "radial" momentum to increase in each direction while conserving the sum.

On the other hand, if the exponent of radial momentum is negative (corresponding to an *accelerated* shock i.e. $\dot{R}(t)$ increases with time), the previous equation predicts a decreasing momentum. It cannot occur on both sides due to the global conservation law, hence this leads to an asymmetry

$$\Pi_+(t) \sim 1/\dot{R}(t) \tag{32}$$

$$\Pi_-(t) = \Pi(0) - \Pi_+(t) \tag{33}$$

where momentum concentrates behind the centre of mass. This situation for elastic particles is in fact very similar to the dissipative blast in one respect: all particles are moving in the same direction and there is no symmetrization of the blast, just like for inelastic particles, although this is due here to masses decreasing with r in such a way that collisions between two particles never lead to a reversal of either particle's velocity. In higher dimensions, dissipative and accelerated conservative blasts are both known to exhibit a shell structure around an empty core [5, 19]: the same non-reversal idea manifests in the fact that, as the innermost particles move toward the shock front, no particle can be reflected by a collision and return to fill the center.

5. Conclusion

We have first considered the case of a one-dimensional system of elastic hard particles in which a large quantity of energy is released at one point. We showed that, unless the repartition of particle masses contains an intrinsic length scale, the wave is effectively zero-dimensional – reducing to the shock front, a single particle in motion.

Adding energy dissipation to the collisions removes this singularity, yet its trace lingers in a transient phase: at first, the front remains solely responsible for the dynamics of the system, but it is not fully constrained by conservation laws, and thus does not obey a simple power-law scaling. Instead, this transient phase displays the logarithmic behaviour previously proposed in experimental studies of granular systems [6, 7]. Once the front becomes coupled to the bulk, the system enters its asymptotic scaling regime, the Momentum-Conserving Snowplow driven by inertial motion, and well-known in astrophysical systems [17].

Both the conservative and dissipative systems analysed in this article demonstrate the role of kinetic insight in deducing scaling behaviours from global symmetries such as conservation laws. Due to the singular nature of the shock front, the correct reasoning may often be obscured if one remains at the hydrodynamic level of description, either in analytical or numerical studies. Further evidence will arise in the study of blasts in higher dimensions, where the core intuitions of this work find a natural generalization [15].

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